Solutions to Problem 1.

a. $\Pr\{R_6 \le 2\} = 1 - e^{-40(1/30)} \approx 0.7364 \qquad (R_6 \sim \text{Exponential}(\lambda = 40))$

b.
$$\begin{aligned}\Pr\{T_{300} \le 6 \mid Y_2 = 150\} &= \Pr\{Y_6 \ge 300 \mid Y_2 = 150\}\\ &= \Pr\{Y_6 - Y_2 \ge 150 \mid Y_2 = 150\}\\ &= \Pr\{Y_6 - Y_2 \ge 150\}\\ &= \Pr\{Y_4 \ge 150\}\\ &= 1 - \sum_{j=0}^{149} \frac{e^{-40\cdot 4}(40\cdot 4)^j}{j!} \approx 0.7956\end{aligned}$$

c.
$$\begin{aligned}E[Y_{16} \mid Y_6 = 250] &= 250 + E[Y_{16} - Y_6 \mid Y_6 = 250]\\ &= 250 + E[Y_{16} - Y_6]\\ &= 250 + E[Y_{10}]\\ &= 250 + 40(10) = 650\end{aligned}$$

d. The first class passengers arrive according to a Poisson process with arrival rate $\lambda_1 = 0.15(40) = 6$.

$$E[T_{1,60}] = \frac{60}{\lambda_1} = 10$$

e.
$$\begin{aligned}\Pr\{Y_{1,1} \ge 5 \text{ and } Y_{1,16} - Y_{1,15} \ge 5\} &= \Pr\{Y_{1,1} \ge 5\}\Pr\{Y_{1,16} - Y_{1,15} \ge 5\} \quad \text{(independent increments)}\\ &= \Pr\{Y_{1,1} \ge 5\}\Pr\{Y_{1,1} \ge 5\} \quad \text{(stationary increments)}\\ &= (1 - \Pr\{Y_{1,1} \le 4\})^2\\ &= \left(1 - \sum_{j=0}^{4} \frac{e^{-6(1)}(6(1))^j}{j!}\right)^2\\ &\approx 0.5111\end{aligned}$$

Solutions to Problem 2.

a. If $0 \le \tau < 6$: $\quad \Lambda(\tau) = \int_0^\tau 10\,dt = 10\tau$

If $6 \le \tau \le 12$: $\quad \Lambda(\tau) = \int_0^6 10\,dt + \int_6^\tau 4\,dt = 4\tau + 36$

$$\Rightarrow \Lambda(\tau) = \begin{cases} 10\tau & \text{if } 0 \le \tau < 6\\ 4\tau + 36 & \text{if } 6 \le \tau \le 12\end{cases}$$

b. $E[Z_5 - Z_2] = \Lambda(5) - \Lambda(2) = 12 \qquad (Z_5 - Z_2 \sim \text{Poisson}(\Lambda(5) - \Lambda(2)) = \text{Poisson}(12))$

c.
$$\begin{aligned}\Pr\{Z_5 - Z_2 \le 15 \mid Z_2 = 30\} &= \Pr\{Z_5 - Z_2 \le 15\}\\ &= \sum_{j=0}^{15} \frac{e^{-12}(12)^j}{j!} \approx 0.8444\end{aligned}$$

Solutions to Problem 3.

In order for an arrival counting process (with arrivals one-at-a-time) to be Poisson, it must satisfy:

- Independent increments: in this context, the number of phone calls arriving at the cell tower in non-overlapping time intervals must be independent
- Stationary increments: in this context, the number of phone calls arriving at the cell tower only depends on the length of the interval, not when the time interval occurs